RAILWAY TURN-OUTS

JACOB M. CLARK.

Mrs

A. D. Barber

"Compliments,"

Jacob W. Colam

A NEW SYSTEM

OF

LAYING OUT

RAILWAY TURN-OUTS

INSTANTLY,

BY

INSPECTION FROM TABLES

BY

JACOB M. CLARK.

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Transport.

PREFATORY NOTE.

THIS little work is issued in its present form by permission of the Publishing Committee of the American Society of Civil Engineers. The text was written in 1869, at which time the tables had assumed nearly their present extent. The author, with a majority of his colleagues on that committee, regarded the matter as too technical to be published in the "Transactions."

The general principle of *sum* and *difference* has long been understood, and is so obvious that many must have hit upon it independently. In 1878 a tract appeared, by Mr. E. A. Giesler, civil engineer and architect, containing a limited set of tables for single turn-outs on a gauge of 4.7, with a peculiar method of applying corrections for curvature in the main track, and for different lengths of switch-bar. This principle is applied, but in a somewhat different way, by William Findlay Shunk, C.E., in "The Field Engineer." A more distinct allusion appears in the last edition (1883) of

Trautwine's "Civil Engineer's Pocket-Book," page 403; and the principle is treated more *in extenso* by J. R. Stephens, following Trautwine, in *Van Nostrand's Magazine*, August, 1883, pages 89–100. Allusions to it are also found in "Field Engineering," by W. H. Searles, C.E., Member of the American Society of Civil Engineers, recently published. These works, each in its proper sphere, are among the most useful hand-books to be found.

It is obvious that the tables apply to "point-switches," so called, by simply substituting for "head-block" the position where the "point-rail" departs from the "angle-rail" by an amount equal to the "throw" of the switch, according to the old method.

Where the frog-angle is less than three and a half degrees it is scarcely practicable to make a firm frog. In rare cases, however, these lower angles may serve to fix the adjustment where, from any cause, it is necessary for one rail to cross another at a very acute angle by means of a pivoted rail.

JACOB M. CLARK.

NEW YORK, September, 1883.

RAILWAY TURN-OUTS.

THE subject of this monograph claims attention, rather on account of the aggregate amount of industry employed in adjusting the contrivances to which it relates, than from its professional range. Numerous and careful treatises attest how fully this claim has been recognized by the profession; the present paper aims to exhibit, in a systematic form, a method of practice, intended, so far as it differs from others, to save time and abridge labor.

The published solutions extant very uniformly regard the turn-out track as located on a curve which is tangent to a *switched* or deflected rail. This multiplies cases, requiring, for exact determination, the construction of diagrams, much calculation, and in general, the use of logarithmic and circular tables. The valuable tables of frog-angles and distances in existence are based on that method, and do not exhibit the corrections sometimes necessary for turn-outs from tracks which are sharply curved.

It is generally more convenient to locate the turn-out upon a curve which is tangent to the main track at a point not far from the heel of the switch. The head-block is then placed

where the departure of the centre lines from each other is equal to the necessary deflection or "throw" of the switch-bar, which, in turn-outs from a straight track, should not be less than half nor more than the entire distance from the head-block back to the tangent-point or point of divergence.

By this device, the exact solutions for all turn-outs (except one of rare occurrence), are reduced to three cases, each of which involves simply the resolution of a right-angle triangle, two of whose parts are known, or of an oblique triangle with three given sides. The same is true of *cross-overs.* They are introduced in this connection to make clear what follows.

TURN-OUTS.

CASE I.—*Turn-out from a Straight Track.*

Let AB (Fig. 1) = the gauge = a,
r' = radius of turn-out track,
m = the degree of curvature of turn-out track,
t = throw of switch,
d = OP = the distance of the head-block from the point of divergence,
D = OF = the distance of the frog,
F = OCF = the frog-angle :

Then,

$$D = \sqrt{2r'a},$$

$$\text{Sin. } F = \frac{r' + \frac{1}{2}a}{\sqrt{2r'a}}, \ \text{tang. } \tfrac{1}{2}F = \sqrt{\frac{a}{2r'}},$$

$$d = \sqrt{2r't},$$

and

$$\text{PF} = D - d = \sqrt{2r'}\,(\sqrt{a} - \sqrt{t}).$$

CASE II.—*Interior Turn-out from a Curved Track.*

(Fig. 2) Let r = radius of main track,
r' = radius of turn-out:
Then the frog-angle F and the distance OF are found by solving the triangle CC′F, whose sides are respectively $(r - \frac{1}{2}a)$, $(r' + \frac{1}{2}a)$, and $(r - r')$.

CASE III.—*Exterior Turn-out from a Curved Track.*

(Fig. 3) The sides of the triangle to be solved are respectively $(r + \frac{1}{2}a)$, $(r' - \frac{1}{2}a)$, and $(r + r')$.

OP is practically the same as if one of the tracks were straight, the other having at the same time a degree of curvature equal to the *difference* of the degrees of curvature of the main and turn-out tracks in Case II., or in Case III. to their *sum*. So that generally (Fig. 4), if

m be the degree of curvature of the main track, m' that of the turn-out, and designating (Fig. 5), $m \pm m'$ as the degree of *divergence* $= M$, we have

$$d = \frac{1}{\sqrt{M}} \times \sqrt{11460t}.$$

These three cases include that of *double turn-outs,* whenever the two branches have equal degrees of divergence from the main track, by simply determining the distance Of (Fig. 6), and the frog-angle f, independently of the main track. The value of OP will be the same, since

$$\frac{1}{\sqrt{2M}} \times \sqrt{11460 \times 2t} = \frac{1}{\sqrt{M}} \times \sqrt{11460t}.$$

Whenever the degrees of divergence from the main track are unequal, it is best, in order that the line of the main track may bisect the departure of the two turn-outs from each other at the head-block, to locate each turn-out from a separate point of divergence. For instance, we may determine the angle and position of the main-frog, F (Fig. 7), belonging to one turn-out, reckoning from its point of divergence, O, as in Case II.; also determine OP. Now let M represent the degree of divergence of the turn-out OB′ from the main track, and M' that of the other turn-out, O′B″. Take

$$\text{PO}' = \text{PO}\sqrt{\frac{M}{M'}}:$$

then O′ will be the point of divergence for the curve O′B″, from which to locate the main frog, F″, and determine its angle.

In the same way, let M'' be the degree of divergence of the two turn-outs from each other. Take

$$PO'' = PO\sqrt{\frac{2M}{M''}},$$

and from O″ determine the position and angle of the central frog, *f*.

The error which arises in this case from the fact that if we consider the curve O′B″ continued backward to O″ it will not be exactly tangent to OB′, though parallel with it, is of no account in practice, for the switch-bar can never conform strictly with the curves.

Triple and multiple turn-outs, diverging through a switch common to them all, have the positions and angles of the interior frogs determined in the same way, by treating the tracks in pairs.

CROSS-OVERS.

The simplest way is to pass from one track to another by a reverse curve.

CASE I.—*Symmetrical Cross-over upon Straight Tracks, with equal Radii.*

(Fig. 8) Let h = the distance of parallel tracks

from centres; also, let r' be the radius, and Q the point of reverse curve.

$$\text{Then } OQ = \sqrt{r'h - \frac{h^2}{4}} = r' \,.\, \sin.\ C.;$$

$$\text{Also, cosin. } C = \frac{r' - \tfrac{1}{2}h}{r'}.$$

CASE II.—*Unsymmetrical Cross-over from Straight Tracks, with unequal Radii.*

(Fig. 9) Let r' and r'' be the radii, at the same time $r' < r''$;

$$\text{Then, cos. } OCQ = \frac{r' + r'' - h}{r' + r''},$$

$$\begin{aligned} OQ &= r' \,.\, \sin.\ C, \\ O'Q &= r'' \sin.\ C, \\ \text{and } OO' &= \sin.\ C\,(r' + r''). \end{aligned}$$

CASE III.—*Cross-over on a Curve.*

(Fig. 10) Let r be the greater radius of parallel tracks.

The sides of the triangle to be resolved are:

$$\left.\begin{array}{l}\text{[a] } (r+r'-h),\ (r-r'),\ \text{and } 2r', \\ \text{[b] } (r+r''-h),\ (r-r'),\ \text{and } (r'+r''),\end{array}\right\}\ \text{if the cross-over has}\ \left\{\begin{array}{l}\text{equal} \\ \text{unequal}\end{array}\right\}\ \text{radii.}$$

The frog-angles, etc., are determined as before explained.

[c] Where the curves are so related that the inside rail of the crossing interferes with the parallel track before reversing, the second frog, F′, has to be determined from O by resolving the triangle whose sides are

$$(r - h + \tfrac{1}{2}a),\ (r - r'),\ \text{and}\ (r' - \tfrac{1}{2}a).$$

NOTE.—Many practitioners prefer to have a short tangent joining the two branches of a cross-over; say traversing the extent between the two frogs. In the case of a symmetrical crossing on straight tracks, this extent, measured along the centre between tracks, is manifestly

$$\frac{(h - a)\cos. F - a}{\sin. F}.$$

If the cross-over is unsymmetrical, or if the main tracks are curved, the expression is more complex and requires a special construction.

GENERAL ANALOGY.

These cases are all connected together by the familiar law that all osculating curves have, at their common tangent-point, their departures from each other absolutely, and near that point approximately, as the squares of the distances or coterminous arcs.

Accordingly, if we assume the one-hundred-feet chain as the unit of linear measure, and substitute for r' its approximate value, $\frac{57.3}{m}$, and for F, $\frac{\sin. F}{\sin. 1^\circ}$, at the same time interpreting M

as the degree of divergence, the expressions in Case I. reduce to

$$D = 1.07\sqrt{\frac{a}{M}} = \frac{\Delta}{\sqrt{M}},$$

$$d = 1.07\sqrt{\frac{t}{M}} = \frac{\delta}{\sqrt{M}},$$

$$\text{and } F = 1.07\sqrt{aM} = \Phi\sqrt{M};$$

in which 1.07 is approximately the length of arc in 100-feet chains, and at the same time the central angle in degrees, corresponding to a versed sine of 1 foot on a 1° curve, while Δ and Φ are the distance and angle of frog on a supposed 1° turn-out from a straight track with a given gauge. This is the simplest form for general approximation, and by stating each case according to its degree of divergence, it will answer for many cases of turn-outs, from curves as well as from straight lines, where minute accuracy is not required. The errors may be much reduced by slightly altering the co-efficients, as shown in the schedule:

Gauges.		4.7		4.83		5.0		6.0		7.0	
	δ when $t = 0.42$.	Δ	Φ	Δ	Φ	Δ	Φ	Δ	Φ	Δ	Φ
Actual values..	69.3	232.1	2° 19′.2	235.3	2° 21′	239.3	2° 24′	262.2	2° 37′.5	283.2	2° 50′
Altered values..	69.3	232.4	2° 18′.6	235.6	2° 20′.3	239.6	2° 23′.1	262.6	2° 36′.7	283.6	2° 49′.1

To make the analogy more correct and complete, D should always be reckoned along the centre of the track which has least curvature.

But the chief value of this expression is that it renders possible the construction of concise tables, from which accurate values can be taken out instantly by inspection, without resort to logarithms or draughtsmanship Where precision is required, in the case of turn-outs from curves, the proper expression is

$$D' = D \pm \mathrm{C},$$
$$F' = F \mp \mathrm{C};$$

in which D and F are the true values for a given turn-out from a straight line, and C a variable correction, due, under the same degree of divergence, to curvature in the main track. These corrections are positive for D and negative for F, when the divergence is interior, and the reverse in the contrary case.

This is the principle of the subjoined tables.

TABLE I. contains a column of degrees and minutes with their corresponding radii, together with the strict values of d, D, and F, for turn-outs from a straight track, under different gauges.

TABLE II. is a condensed table of corrections, to be applied according to the algebraic sign at the top of each column, when the divergence is interior from a curved track.

TABLE III. Like corrections for exterior divergences.

As these corrections are suited for a gauge of 4.7, they have to be changed for other gauges. The multipliers in the margin are sufficiently correct for the purpose.

A single numerical example may illustrate their use:

Let it be required to pass from a 4° curve to an interior parallel track, by a 12° reverse curve (Fig. 10); the gauge being 7 feet, and the parallel tracks 13.4 feet apart from centres.

It is manifest from the figure that the point of reverse curve, Q, divides both the distance between centres, and the whole extent of the cross-over, O‴ O′, very nearly in the inverse ratio of the degrees of divergence; in this case as 16 to 8 or as 2 to 1.

The radius of the centre between tracks is 1426 feet, and its degree of curvature (by interpolation) = 4° 01′.

In like manner the degree of curvature of the inside track is 4° 02′.

We have, then, for the frog F, $M = 8°$; and from the tables,

	d	D	$D-d$	F
	25.1	100.1		8° 00′
(correction from Table II.) × 1.5 =		+ .3		− 02
Result.........		100.4	75.3	7°.58′.

For F' we have to reckon the degree of di-

vergence from O‴, along the centre between tracks.

The degree of curvature for $r - \frac{1}{2}h =$ 471.6 is........................		12° 10′,
That for centre between tracks.....	=	4 01
Wherefore, M'	=	8° 09′,

and $h - a = (13.4 - 7.0) = 6.4$.

Therefore, for F',	D	F'
	94.7	7° 43′
(correction from Table II.)$\times 1.3 =$	$+.3$	$-$ 01
Result	95.0	7° 42′.

To determine O‴Q and QO′, we first divide $h = 13.4$, in the proportion of 16 to 8. The parts are 8.93 and 4.47.

Suppose $M = 16°$, and $a = 4.47$, we have by interpolation,

$$QO' = 56.3\text{, and } O'P' = 17.7,$$
$$QO''' = 2QO' = 112.6,$$

wherefore, $O'''O' = 168.9$,

and the distance between head-blocks,

$$PP' = (56.3 - 17.7) + (112.6 - 25.1) = 126.1\,,$$

which may be reckoned along the centre between tracks.

Suppose, in addition, that at P (Fig. 7) is a double switch, diverging exteriorly on a 12° curve.

Then $M = 16°$, and we have from the tables,

	d	D	$D-d$		F
	17.7	70.9			11° 16′
(corrections from Table III.) × 1.5		— .2		+	02
		70.7			
Result.... PF″ =			53.0,		$F'' =$ 11° 18′.

For the centre frog, f, we have $M = 24°$; and from the tables,

	$D =$ 58.0,		$F =$ 13° 45′,
(correction from Table III.) × 1.5..........	— .5	+	05
	57.5		
PO″ = d for 12°..	20.5		
Result.........Pf =	37.0		$f =$ 13° 50′.

The entire work takes less time than it would to prepare the case for solution by trigonometry.

The degree of divergence is useful in a variety of ways for solving minor details in the field. For instance:

[1] In the case just cited we may locate the point of reverse curve, and also decide to which branch the frog F′ (Fig. 10) belongs, by laying a line AB (Fig. 11) transversely joining the right (or left) hand rails of the parallel tracks,

and laying off AD and CB as M' to M, and joining CD. If the intersection E falls within the parallel track, both frogs belong to the same branch; and if we make Eq $= \frac{1}{2}a$, the point of reverse curve will be in the parallel of q.

[2] In a double turn-out, to align the centre frog.

At the proper distance from the head-block for the point of the frog (Fig. 12), divide the gauge AB in *inverse* proportion to the degrees of divergence, so that $\frac{\text{Bf}}{\text{Af}} = \frac{M'}{M}$. The point of the frog will be at f. Now divide the base of the frog in *direct* proportion, making $\frac{\text{bc}}{\text{ab}} = \frac{M}{M'}$, and lay the frog so that f b will be parallel with the main track.

[3] It also enables us to lay off one curve from another by ordinates, precisely as we would lay off a curve from its tangent.

[4] If the proposition is to locate a cross-over upon two given frogs, the conditions of the case may be settled in this way:

[a] If the frogs are *alike*, the *degrees* of *divergence* are the same, and the reversing point bisects the whole *extent* of the cross-over, and is in the centre between tracks.

[b] If they are unlike, and their angles differ in a *greater ratio* than that of the *square roots* of the gauge and the "virtual" gauge between tracks (clear distance between tracks plus twice

the rail-head), they cannot be correctly laid in the same cross-over.

[c] If they differ in a *less ratio*, they belong to *different* branches of the reverse curve.

[d] If in that *precise ratio*, they belong to the *same* branch, and the reversing point may be opposite the farthest frog, or beyond it, as convenient.

COMPOUND DIVERGENCE.

The only case which eludes the direct application of the tables is where a tangent point, or a point of compound or reverse curvature, occurs in the main track, between the switch and the frog, forming a *compound* divergence. Sometimes a case of the kind may be reduced by so compounding the turn-out as to maintain the same degree of divergence throughout; but, in general, a special adjustment is better, by one of two methods.

The *first* requires an alteration in the main track, and depends on a problem of frequent occurrence in "location," not found in the manuals.

PROBLEM.—*To join unequal tangents by two arcs of equal length, or rather, having equal chords.*

Let AT and TD (Fig. 13) be the tangents. Take TB′ = AT, also TB = TD, and join AD

and AB′. Make the angle GAD = DAB′. Bisect the angle TAG by the indefinite line AH, and cut this line in E, by a perpendicular to AD, from its middle point O. From E draw EC′ indefinitely, at right angles with AG; also, draw DC at right angles with AT; they will intersect EC′ in C′ and C″, which will be the centres of the two arcs required, tangent to each other in E. Their degrees of curvature are respectively,

$$m' = \frac{\text{TAB}' - 2\text{DAB}'}{\text{length of arc AE}},$$

and

$$m'' = \frac{\text{TAB} + 2\text{DAB}'}{\text{length of arc ED}}.$$

The truth of this construction is proved by drawing DB parallel with AB′, and also the radii BC and DC; for by comparing the angles involved, the triangles AC′E and EC″D are found to be isosceles.

The solution would be the same (changing the signs), if A were forward of B, or the problem reversed and considered from D.

Whenever 2DAB′ is greater than TAB′, *m* becomes negative, and E falls the other side of the tangent AT, and is therefore a point of reverse curvature.

It is manifest also that the solution includes the case where B is a point of either reversed or compound curvature, since the angle TAB′

is half the angle formed by the tangents, produced from A and D.

The operation is instantly performed in the field, with sufficient accuracy, by placing the instrument at A and observing the angle DAB′ (TAB′ being already known), and tracing the curve AE, half way from A to B, with a degree of curvature $m' = \frac{\text{TAB}' - 2\text{DAB}'}{\text{AE}}$, and completing the branch ED with a degree of curvature $m'' = \frac{\text{TAB}' + 2\text{DAB}'}{\text{ED}}$.

It may be done with equal facility by ordinates, whenever AB is a straight line, and BQD a circular arc joining the tangents, by producing the curve backward to a point N, opposite A, and measuring AN, and also AK to the prolongation of the chord BD. The ordinate EQ is then $= \frac{1}{4}$ (AK — AN); and the branch AE may be set off from the tangent AT, and the branch DE from the curve DQ, by ordinates proportional to the squares of the distances from A and D respectively.

The degree of curvature of AE is $m' = 11460 \left(\frac{\text{ZE}}{\text{AE}^2}\right)$; that is, ZE divided by the middle ordinate of a 1° curve on a chord = 2AE.

Whenever, therefore, in the case supposed, A is to be the origin of a double or multiple turn-out, we may fix a point D in the main track, a little farther from A than twice the probable distance of the farthest frog; and by compound-

ing the main track in the manner just explained, all the divergences become simple, and resolvable by the tables.

The *Second* method is more simple, and is always preferable when A is the origin of a single turn-out, since it involves no alteration in the main track. The principle is the same, whether the divergence be interior or exterior.

If we consider the frog at F already located, it is evident that the frog-angle (Fig. 14) is, in the former case, greater, and in the latter case less (Fig. 15), than would be required for a *simple* divergence of the same degree. Now, suppose both curves continued to some point O‴, where they become parallel; and designate the distance AB as k, and the distance of the curves from each other at O‴ as e, we shall have, in the first case,

$$BO''' = k \times \frac{r}{r - r'}, \text{ and } e = \tfrac{1}{2} \times \frac{k^2}{r - r'}, \text{ (Fig. 14),}$$

and in the second,

$$BO''' = k \times \frac{r}{r + r'}, \text{ and } e = \tfrac{1}{2} \times \frac{k^2}{r + r'}, \text{ (Fig. 15);}$$

or universally, adhering to our former notation:

$$BO''' = k \times \frac{m'}{M};$$

and

$$e = \frac{k^2}{11460} \times \frac{mm'}{M};$$

that is,

$e =$ *the middle ordinate of a curve whose degree of curvature is* $\frac{mm'}{M}$, on a chord $= 2k$.

Now, if we conceive one or both the curves to be displaced along the normal CC′, so as to become tangent to each other at O‴, or, which is the same thing in practice, describe a new pair of curves from the centres C and C‴, passing through O‴, the frog-angle at F and its distance from O‴ are manifestly such as would be required for the degree of divergence M, and a gauge equal in the former case to $a + e$, and in the latter to $a - e$; and we may therefore, having found O‴, and the hypothetical gauge $a + e$ or $a - e$, as just explained, locate the frog from O‴, and determine its angle, by interpolation, from the tables.

The head-block P may then be located from the real point of divergence A, as before explained, according to the degree of divergence which subsists from A to B.

For example: if B were the tangent point of a 4° curve, and it were required to depart by an interior turn-out on a 12° curve from A, 35 feet back of B, the gauge being 4.7, we should have by the formula,

$$\text{BO}''' = 35 \times \frac{12}{8} = 52.5;$$

also,

e = ordinate for $\left(\frac{12 \times 4}{8}\right)^\circ$ curve on 70-feet chord, $= 0.64$;

whence, $a + e = 4.7 + 0.64 = 5.34$,

and from the tables, M being 8°, with gauge = 5.34,

$$O'''F = 87.3 \text{ and } F = 6^\circ\ 58',$$

and also,

$$\text{when } M = 12^\circ,\ AP = 20.5;$$

whence,

$$PF = 87.3 - (52.5 - 35) - 20.5 = 49.3.$$

This construction, like the former, applies equally well when B is a point of either compound or reverse curvature; and is applicable to multiple turn-outs, by considering the tracks in pairs.

GENERAL DIRECTIONS FOR USING THE TABLES.

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1. To find the proper frog-angle and distance for a turn-out of given radius, or degree of curvature.

If the main track is straight, use this degree of curvature as the *degree of divergence;* if curved the same way as the turn-out, use the

difference of the degrees of curvature; if the opposite way, their *sum.*

Find the proper gauge at the top of Table I., and follow the double column down to opposite the required degree of divergence at the left hand of the page: under the letter *F* is the frog-angle, and under *D* its distance from the *point* of *divergence* or tangent point of the turn-out; also, near the left hand, under *d*, the distance from the tangent point to the head-block, for different throws of switch (which see at the top). The difference of *D* and *d* is the distance from head-block to frog, *measured along the centre line.*

N.B. The distances 6.4, 7.4, 8.4, etc., are "virtual" gauges (clear distance between tracks plus twice the rail-head), for cross-overs so conditioned that both frogs belong to the same branch of the reverse curve.

Correction.—If the main track is so sharply curved as to vitiate the result, look along the top of Table II. or III. (according as the divergence is "interior" or "exterior") for the figure nearest the *degree of curvature* of the *main track.* Under this, and opposite the figure nearest the *degree of divergence*, are the *corrections*, in feet and decimals for the distance (headed D), and in minutes for the frog-angle (headed F). D is to be added to the distance, and F subtracted from the frog-angle, when the divergence is "interior," and the reverse in the contrary case.

Adjust the corrections according to gauge before applying them.

2. To lay out a turn-out when the frog-angle is given.

Under the proper gauge and letter F, find the frog-angle; opposite are the distances required, as before, and the degree of divergence. The latter, when compared with the degree of curvature of the main track, will indicate the necessary *curvature* of the turn-out. If the main track is sharply curved, the frog-angle will indicate the degree of divergence, sufficiently near to find the *corrections* from Table II. or III. If "interior," *add* the correction to the frog-angle; if "exterior," *subtract* it; then the frog-angle so modified will give from Table I. the *true* degree of divergence; the distance to be corrected as above explained.

3. In a double turn-out, to find the distance of the centre frog from the head-block, deduct from the whole distance *D*, corresponding to the degree of *divergence* of the *two turn-out tracks* as to *each other*, a distance, *d*, corresponding to *half* that degree.

HINTS TO TRACK-MASTERS.

1. *To measure the angle of a frog.*

From the point of the frog, with a radius of $57\frac{3}{10}$ inches, strike an arc of a circle across the frog; the length of this arc in inches and 60ths of an inch is the frog-angle in degrees and minutes.

2. *To measure the degree of curvature of a track already laid.*

On a well-lined portion, measure the middle ordinate in feet on a 200-feet chord; increase this ordinate by $\frac{1}{7}$ of itself, the result expresses the degree of curvature.

Or: use the middle ordinate of a 214-feet chord, or twice the ordinate on $151\frac{3}{10}$ feet, without alteration.

3. *To find the tangent point.*

Find where the rail has departed from the range of the tangent by a distance in feet and 60ths equal to the degree of curvature in degrees and minutes; the tangent point is 107 feet back.

4. *To calculate the middle ordinate for any chord.*

Multiply the square of half the chord in feet by the degree of curvature, diminish the product by $\frac{1}{8}$ of itself, and set the decimal point four places to the left.

TABLE I.

Degree of Curvature or Divergence.	Corresponding Radius.	Distance from Origin to Head-Block. $t=0.42$	$t=0.44$	$t=0.46$	Distances of Frog from Origin, and Frog-Angles. When a or $k-a=$ 4.7 D.	4.7 F.	4.83 D.	4.83 F.	5.0 D.	5.0 F.	5.5 D.	5.5 F.	6.0 D.	6.0 F.	6.4 D.	6.4 F.	7.0 D.	7.0 F.	7.4 D.	7.4 F.	8.4 D.	8.4 F.
1° ′	5729.7	69.3	70.9	72.5	232.1	2° 19′	235.3	2° 21′	239.3	2° 24′	251.3	2° 31′	262.2	2° 37′	270.8	2° 42′	283.2	2° 50′	291.7	2° 55′	310.2	3° 06′
10	4911.2	64.1	65.6	67.	214.9	30	227.8	33	221.6	35	232.7	42	242.8	50	250.7	55	262.2	3 03	271.0	3 08	287.2	21
20	4297.3	60.	61.4	62.8	201.	41	203.7	43	206.3	47	217.6	54	227.1	3 01	234.5	3 07	245.2	16	252.3	22	268.6	35
30	3819.8	56.6	57.9	59.2	189.5	51	192.1	53	194.4	57	205.1	04	224.1	12	221.1	19	231.2	28	237.9	34	253.3	48
40	3437.9	53.7	54.9	56.2	179.8	3	182.3	3 02	184.4	3 06	194.5	14	213.1	23	209.8	30	219.3	39	225.6	46	240.4	4
50	3125.4	51.2	52.4	53.6	171.4	08	173.8	11	175.8	15	185.5	24	203.7	33	200.	40	209.1	50	215.1	57	229.2	12
2	2864.9	49.	50.2	51.3	164.1	17	166.4	19	169.3	23	177.5	33	185.4	42	191.5	50	200.2	4	205.9	4 07	219.3	23
10	2644.6	47.1	48.2	49.3	157.7	25	159.9	27	162.7	31	170.6	42	178.2	51	184.	59	192.3	10	196.7	18	210.7	34
20	2455.7	45.4	46.4	47.5	151.9	33	154.1	35	156.7	39	164.4	50	171.7	4	177.3	4 08	185.3	20	190.5	28	203.1	44
30	2292.0	43.8	44.8	45.9	146.8	40	148.8	43	150.4	47	158.8	58	165.8	09	171.3	17	179.	29	184.	37	196.2	54
40	2148.8	42.4	43.4	44.4	142.1	47	144.1	50	146.6	54	153.7	4 06	160.6	17	165.8	25	173.3	37	178.1	45	190.	5 04
50	2022.4	41.2	42.1	43.1	137.9	54	139.8	57	142.2	4 02	149.1	13	155.7	25	160.9	33	168.2	46	172.9	54	184.4	13
3	1910.1	40.	40.9	41.9	134.	4 01	135.9	4 04	138.2	09	144.9	21	151.3	32	156.3	41	163.5	54	168.1	5 03	179.1	22
10	1809.6	38.9	39.8	40.7	130.4	08	132.3	11	134.5	16	141.1	28	147.4	40	152.2	49	159.1	5 03	163.6	12	174.3	31
20	1719.1	38.	38.8	39.7	127.1	15	128.9	18	131.1	23	137.5	35	143.6	48	148.3	56	155.1	11	159.5	20	169.9	40
30	1637.3	37.	37.9	38.8	124.1	21	125.8	25	128.	29	134.2	42	140.2	55	144.6	5 04	151.4	18	155.7	27	165.8	48
40	1562.9	36.2	37.	37.9	121.2	27	122.9	31	125.	35	131.1	49	136.9	5 02	141.4	11	147.9	26	152.1	35	162.	56
50	1495.0	35.4	36.2	37.	118.6	33	120.2	37	122.3	41	128.2	55	134.	08	138.3	18	144.6	33	148.7	42	158.5	6 04
4	1432.7	34.6	35.4	36.3	116.1	38	117.7	42	119.7	47	125.5	5 01	131.1	15	135.4	25	141.6	40	145.6	49	155.1	12
10	1375.4	33.9	34.7	35.5	113.7	44	115.3	48	117.3	53	123.	07	128.5	21	132.7	31	138.7	47	142.6	56	152.	19
20	1322.5	33.3	34.1	34.8	111.5	49	113.1	54	115.	59	120.6	13	126.	28	130.1	38	136.	54	139.9	6 03	149.	27
30	1273.6	32.7	33.4	34.2	109.4	55	110.9	5	112.9	5 05	118.3	19	123.7	34	127.6	44	133.5	6	137.3	10	146.3	34
40	1228.1	32.1	32.8	33.6	107.4	5	108.9	05	110.9	10	116.2	25	121.5	40	125.3	50	131.1	07	134.8	17	143.6	41
50	1185.7	31.5	32.2	33.	105.6	06	107.	10	108.9	16	114.2	31	119.4	46	123.2	57	128.8	13	132.5	23	141.2	49

TABLE I.—*Continued.*

Degree of Curvature or Divergence.	Corresponding Radius.	Distance from Origin to Head-Block.			Distances of Frog from Origin, and Frog-Angles. When a or $h-a=$																	
					4.7		4.83		5.0		5.5		6.0		6.4		7.0		7.4		8.4	
		$t=$ 0.42	$t=$ 0.44	$t=$ 0.46	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.
5° ′	1146.3	31.	31.7	32.4	103.8	5° 11′	105.2	5° 16′	107.1	5° 21′	112.2	5° 37′	117.3	5° 52′	121.1	6° 03′	126.6	6° 20′	130.2	6° 30′	138.8	6° 55′
10	1109.3	30.5	31.2	31.9	102.1	16	103.5	21	105.3	26	110.4	42	115.5	58	119.1	10	124.6	26	128.2	36	136.5	7 02
20	1074.7	30.	30.7	31.4	100.5	21	101.9	26	103.7	32	108.7	48	113.6	6 03	117.2	15	122.6	32	126.1	42	134.4	09
30	1042.3	29.5	30.2	30.9	99.	26	100.3	31	102.1	37	107.	53	111.9	09	115.5	20	120.7	38	124.1	48	132.3	16
40	1011.5	29.1	29.8	30.5	97.5	31	98.9	36	100.6	42	105.5	58	110.3	15	113.8	26	118.9	44	122.3	55	130.3	22
50	982.6	28.7	29.4	30.	96.1	36	97.4	41	99.1	47	104.	6 03	108.7	20	112.1	32	117.2	50	120.5	7 01	128.5	29
6	955.4	28.3	28.9	29.6	94.8	41	96.1	45	97.7	51	102.5	08	107.2	25	110.5	37	115.6	56	118.8	07	126.7	35
10	929.6	27.9	28.6	29.2	93.5	45	94.8	50	96.4	56	101.1	13	105.7	31	109.	43	114.	7 01	117.2	12	125.	41
20	905.1	27.5	28.2	28.8	92.2	50	93.5	55	95.1	6 01	99.8	18	104.3	36	107.6	49	112.5	07	115.7	18	123.3	47
30	882.	27.2	27.8	28.4	91.	55	92.3	59	93.9	06	98.5	23	103.	41	106.2	53	111.1	12	114.2	24	121.8	55
40	859.9	26.8	27.5	28.1	89.9	59	91.1	6 04	92.7	10	97.2	28	101.7	46	104.9	58	110.7	18	112.8	30	120.3	8
50	839.	26.5	27.1	27.7	88.8	6 04	90.	09	91.6	15	96.	33	100.4	51	103.6	7 03	109.3	23	111.3	35	119.8	06
7	819.	26.2	26.8	27.4	87.7	08	89.	13	90.5	20	94.9	38	99.2	56	102.3	09	107.	29	110.	41	117.4	11
10	800.	25.9	26.5	27.1	86.7	12	87.9	17	89.4	24	93.8	43	98.1	7 01	101.1	14	105.8	34	108.8	46	116.0	17
20	781.8	25.6	26.2	26.8	85.7	17	86.9	22	88.4	28	92.7	47	96.9	06	100.	19	104.6	39	107.5	51	114.6	23
30	764.5	25.3	25.9	26.5	84.8	21	85.9	26	87.4	33	91.7	52	95.8	11	98.9	24	103.4	44	106.3	56	113.3	29
40	747.9	25.	25.6	26.2	83.8	25	85.	30	86.5	37	90.7	56	94.8	15	97.8	29	102.3	50	105.2	8 03	112.1	35
50	732.	24.8	25.3	25.9	82.9	29	84.1	35	85.5	41	89.7	7 01	93.8	20	96.7	34	101.2	55	104.	08	110.9	40
8	716.8	24.5	25.1	25.6	82.1	34	83.2	39	84.6	46	88.8	05	92.8	25	95.8	39	100.1	8	102.9	13	109.7	45
10	702.2	24.3	24.8	24.4	81.3	37	82.4	43	83.8	50	87.9	10	91.8	29	94.7	43	99.1	05	101.9	18	108.6	50
20	688.2	24.	24.6	25.1	80.4	41	81.5	47	82.9	54	87.	14	90.9	34	93.9	48	98.1	09	100.9	23	107.5	56
30	674.7	23.8	24.3	24.9	79.6	45	80.7	51	82.1	58	86.2	18	90.	38	92.9	53	97.2	14	99.9	28	106.5	9 01
40	661.7	23.5	24.1	24.6	78.9	49	80.	55	81.3	7 02	85.3	23	89.1	42	92.1	57	96.2	19	98.9	33	105.4	06
50	640.3	23.2	23.8	24.4	78.1	53	79.2	59	80.6	06	84.5	27	88.3	47	91.2	8 02	95.2	24	97.0	38	104.4	12

TABLE I.—*Continued.*

Degree of Curvature or Divergence.	Corresponding Radius.	Distance from Origin to Head-Block. $t=0.42$	$t=0.44$	$t=0.46$	Distances of Frog from Origin, and Frog-Angles. When a or $h-a=$ 4.7 D.	4.7 F.	4.83 D.	4.83 F.	5.0 D.	5.0 F.	5.5 D.	5.5 F.	6.0 D.	6.0 F.	6.4 D.	6.4 F.	7.0 D.	7.0 F.	7.4 D.	7.4 F.	8.4 D.	8.4 F.
9° ′	637.3	23.1	23.6	24.2	77.4	6° 57′	78.5	7° 03′	79.8	7° 10′	83.7	7° 31′	87.5	7° 51′	90.9	8° 06′	94.5	8° 29′	97.1	8° 43′	103.5	9° 17′
10	625.7	22.8	23.4	24.	76.7	7 01	77.7	07	79.1	14	83.	35	86.6	55	89.5	11	93.6	33	96.2	47	102.5	22
20	614.6	22.7	23.2	23.7	76.	05	77.	11	78.4	18	82.2	39	85.9	59	88.7	15	92.7	38	95.3	52	101.6	27
30	603.8	22.5	23.	23.5	75.4	09	76.4	14	77.6	22	81.5	43	85.1	8 04	87.9	20	91.9	42	94.5	56	100.7	32
40	593.4	22.3	22.8	23.3	74.7	12	75.7	18	77.	26	80.8	47	84.4	08	87.2	24	91.1	47	93.7	9 01	99.9	37
50	583.4	22.1	22.6	23.1	74.1	16	75.1	22	76.4	29	80.1	51	83.7	12	86.4	29	90.4	51	92.9	06	99.	42
10	573.7	21.9	22.4	22.9	73.4	19	74.4	25	75.7	33	79.4	55	83.	16	85.7	33	89.6	56	92.1	11	98.2	47
10	564.3	21.7	22.2	22.7	72.8	23	73.8	29	75.1	37	78.8	59	82.3	20	85.	36	88.9	9	91.4	15	97.4	51
20	555.2	21.6	22.	22.6	72.3	27	73.3	33	74.5	41	78.2	8 03	81.6	25	84.3	41	88.2	05	90.7	20	96.6	56
30	546.4	21.4	21.9	22.4	71.7	30	72.7	36	73.9	44	77.5	07	81.	29	83.6	45	87.5	09	90.	24	95.8	10 01
40	537.9	21.2	21.7	22.2	71.1	34	72.1	40	73.3	48	76.9	11	80.3	33	83.	49	86.7	13	89.3	28	95.1	06
50	529.7	21.1	21.5	22.	70.6	37	71.5	43	72.8	51	76.3	14	79.7	37	82.3	53	86.	18	88.6	33	94.4	10
11	521.7	20.9	21.4	21.9	70.	41	71.	47	72.2	55	75.8	18	79.1	40	81.7	57	85.5	22	87.9	38	93.7	15
10	513.9	20.7	21.2	21.7	69.5	44	70.5	51	71.7	59	75.2	22	78.5	44	81.1	9 01	84.8	26	87.2	42	92.9	21
20	506.4	20.6	21.1	21.5	69.	48	69.9	54	71.2	8 02	74.6	26	77.9	48	80.5	05	84.2	30	86.6	46	92.3	26
30	499.1	20.4	20.9	21.4	68.5	51	69.4	58	70.6	06	74.1	29	77.4	52	79.9	09	83.6	34	85.9	50	91.6	21
40	492.	20.3	20.8	21.2	68.	54	68.9	8 01	70.1	09	73.6	33	76.8	56	79.3	13	83.	39	85.3	55	90.9	34
50	485.1	20.1	20.6	21.1	67.5	58	68.5	04	69.6	13	73.1	37	76.3	9	78.8	17	82.4	43	84.7	59	90.3	38
12	478.3	20.	20.5	20.9	67.1	8 01	68.	08	69.2	16	72.5	40	75.8	04	78.3	22	81.8	47	84.1	10 03	89.7	42
10	471.8	19.9	20.3	20.8	66.6	04	67.5	11	68.7	20	72.	44	75.3	07	77.8	25	81.3	51	83.5	07	89.	46
20	465.5	19.7	20.2	20.7	66.2	08	67.1	14	68.2	23	71.6	47	74.8	11	77.1	29	80.7	55	83.	11	88.4	51
30	459.3	19.6	20.1	20.5	65.7	11	66.6	18	67.8	26	71.1	51	74.3	15	76.7	33	80.2	59	82.4	15	87.9	55
40	453.3	19.5	19.9	20.4	65.3	14	66.2	21	67.3	30	70.6	54	73.8	18	76.2	37	79.7	10 03	81.8	19	87.2	11
50	447.4	19.3	19.8	20.2	64.9	17	65.8	24	66.9	33	70.2	58	73.3	22	75.7	41	79.2	07	81.3	23	86.7	04

TABLE I.—*Continued.*

Degree of Curvature or Divergence.	Corresponding Radius.	Distance from Origin to Head-Block.			Distances of Frog from Origin, and Frog-Angles. When a or $h-a$ = 4.7		4.83		5.0		5.5		6.0		6.4		7.0		7.4		8.4	
		$t=0.42$	$t=0.44$	$t=0.46$	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.
13° ′	441.7	19.2	19.7	20.1	64.4	8° 21′	65.3	8° 28′	66.5	8° 36′	69.7	9° 01′	72.8	9° 25′	75.3	9° 44′	78.6	10° 10′	80.8	10° 27′	86.2	11° 08′
10	436.1	19.1	19.5	20.	64.	24	64.9	31	66.	40	69.4	05	72.4	29	74.7	48	78.2	14	80.3	31	85.7	13
20	430.7	19.	19.4	19.9	63.6	27	64.5	34	65.6	43	68.9	08	71.9	33	74.3	51	77.7	18	79.8	35	85.1	17
30	425.4	18.9	19.3	19.7	63.2	30	64.1	37	65.2	46	68.5	12	71.5	36	73.8	55	77.2	22	79.3	38	84.6	21
40	420.2	18.7	19.2	19.6	62.9	33	63.7	40	64.8	49	68.1	15	71.	40	73.4	58	76.7	26	78.8	42	84.1	25
50	415.2	18.6	19.1	19.5	62.5	36	63.3	43	64.4	52	67.7	18	70.6	43	72.9	10 02	76.2	30	78.3	46	83.6	30
14	410.3	18.5	19.	19.4	62.1	39	63.	47	64.1	56	67.2	22	70.2	47	72.5	06	75.8	33	77.9	50	83.	33
10	405.5	18.4	18.8	19.3	61.7	42	62.6	50	63.7	59	66.8	25	69.7	50	72.1	09	75.3	37	77.4	54	82.6	37
20	400.8	18.3	18.7	19.2	61.4	45	62.2	53	63.3	9 02	66.4	28	69.3	53	71.6	13	74.9	41	77.	58	82.1	41
30	396.2	18.2	18.6	19.	61.	48	61.9	56	62.9	05	66.	31	68.9	57	71.2	17	74.5	44	76.5	11 01	81.6	45
40	391.7	18.1	18.5	18.9	60.7	51	61.5	59	62.6	08	65.6	35	68.5	10	70.8	20	74.1	48	76.	05	81.2	49
50	387.3	18.	18.4	18.8	60.3	54	61.2	9 02	62.2	11	65.3	38	68.2	04	70.4	24	73.6	51	75.6	09	80.7	53
15	383.1	17.9	18.3	18.7	60.	57	60.9	05	61.9	14	64.9	41	67.8	07	70.	27	73.2	55	75.2	13	80.2	57
10	378.9	17.8	18.2	18.6	59.7	9	60.5	08	61.6	17	64.6	44	67.4	10	69.6	31	72.8	59	74.8	17	79.8	12 02
20	374.8	17.7	18.1	18.5	59.4	03	60.2	11	61.2	20	64.2	47	67.1	14	69.3	34	72.4	11 02	74.4	20	79.4	06
30	370.8	17.6	18.	18.4	59.	06	59.9	14	60.9	23	63.9	51	66.7	17	68.9	37	72.1	06	74.	24	78.9	09
40	366.9	17.5	17.9	18.3	58.7	09	59.5	17	60.6	26	63.5	54	66.3	20	68.5	41	71.7	09	73.6	28	78.5	13
50	363.	17.4	17.8	18.2	58.4	12	59.2	20	60.2	29	63.2	57	65.9	23	68.2	44	71.3	13	73.2	31	78.1	17
16	359.3	17.3	17.7	18.1	58.1	15	58.9	23	59.9	32	62.9	10	65.7	27	67.8	47	70.9	16	72.8	35	77.7	21
10	355.6	17.2	17.6	18.	57.8	18	58.6	25	59.6	35	62.5	03	65.3	30	67.4	51	70.6	20	72.4	38	77.3	24
20	352.	17.1	17.5	17.9	57.5	21	58.3	28	59.3	39	62.2	06	65.	33	67.1	53	70.2	23	72.	42	76.9	28
30	348.5	17.1	17.5	17.9	57.2	23	58.	31	59.	41	61.9	09	64.7	36	66.8	57	69.8	27	71.7	45	76.5	32
40	345.	17.	17.4	17.8	56.9	26	57.7	34	58.7	44	61.6	12	64.3	39	66.4	11	69.5	30	71.3	49	76.1	36
50	341.6	16.9	17.3	17.7	56.7	29	57.4	37	58.3	47	61.3	15	64.	43	66.1	04	69.2	35	71.	52	75.7	40

TABLE I.—*Continued.*

Degree of Curvature or Divergence.	Corresponding Radius.	Distance from Origin to Head-Block. $t=0.42$	$t=0.44$	$t=0.46$	Distances of Frog from Origin, and Frog-Angles. When a or $h-a=$ 4.7 D.	4.7 F.	4.83 D.	4.83 F.	5.0 D.	5.0 F.	5.5 D.	5.5 F.	6.0 D.	6.0 F	6.4 D.	6.4 F.	7.0 D.	7.0 F.	7.4 D.	7.4 F.	8.4 D.	8.4 F.
17° ′	338.3	16.8	17.2	17.6	56.4	9° 31′	57.2	9° 40′	58.2	9° 50′	61.	10° 18′	63.7	10° 46′	65.8	11° 07′	68.8	11° 37′	70.7	11° 56′	75.4	12° 43′
10	335.	16.7	17.1	17.5	56.1	34	56.9	42	57.9	52	60.7	21	63.4	49	65.4	10	68.5	40	70.4	59	75.	46
20	331.8	16.6	17.	17.4	55.8	37	56.6	45	57.6	55	60.4	24	63.1	52	65.1	14	68.2	44	70.1	12 03	74.6	50
30	328.7	16.6	17.	17.3	55.6	40	56.4	48	57.3	58	60.1	27	62.8	55	64.9	17	67.8	47	69.8	07	74.3	54
40	325.6	16.5	16.9	17.3	55.3	43	56.1	51	57.1	10 01	59.8	30	62.5	58	64.6	20	67.5	50	69.4	10	73.9	57
50	322.6	16.4	16.8	17.2	55.1	45	55.8	53	56.8	04	59.6	33	62.2	11 01	64.3	23	67.2	54	69.1	13	73.6	13 01
18	319.6	16.3	16.7	17.1	54.8	48	55.6	56	56.5	07	59.3	36	61.9	04	64.	26	66.9	57	68.8	17	73.3	05
10	316.7	16.3	16.6	17.	54.6	51	55.3	59	56.3	09	59.	39	61.7	07	63.7	30	66.6	12	68.5	20	73.	09
20	313.9	16.2	16.6	16.9	54.3	53	55.1	10 02	56.	12	58.7	42	61.4	10	63.5	33	66.3	03	68.2	24	72.7	12
30	311.1	16.1	16.5	16.9	54.1	56	54.8	04	55.8	15	58.5	45	61.1	13	63.	36	66.	07	67.9	27	72.3	16
40	308.3	16.	16.4	16.8	53.8	59	54.6	07	55.5	17	58.2	47	60.8	16	62.9	39	65.7	10	67.6	30	72.	19
50	305.6	16.	16.3	16.7	53.6	10 01	54.3	11	55.3	20	58.	50	60.6	19	62.6	42	65.4	13	67.3	34	71.7	22
19	302.9	15.9	16.3	16.6	53.4	04	54.1	12	55.	23	57.7	53	60.3	22	62.3	45	65	16	67.	37	71.3	26
10	300.3	15.8	16.2	16.6	53.1	07	53.9	15	54.8	26	57.5	56	60.	25	62	48	64.8	19	66.7	40	71.1	29
20	297.8	15.8	16.1	16.4	52.9	09	53.6	17	54.6	28	57.2	59	59.8	28	61.8	51	64.6	23	66.4	43	70.8	33
30	295.3	15.7	16.1	16.4	52.7	12	53.4	20	54.3	31	57.	11 01	59.5	31	61.5	54	64.3	26	66.2	46	70.5	36
40	292.8	15.6	16.	16.4	52.5	14	53.2	23	54.1	33	56.8	04	59.3	34	61.2	57	64.	29	65.9	50	70.2	40
50	290.3	15.6	15.9	16.3	52.2	17	53.	25	53.9	36	56.5	07	59.	36	61.	12	63.8	32	65.6	53	69.9	43
20	287.9	15.5	15.9	16.2	52.	19	52.7	28	53.7	39	56.3	10	58.8	39	60.7	03	63.5	35	65.3	56	69.6	46
10	285.7	15.4	15.8	16.2	51.8	22	52.5	30	53.4	41	56.1	12	58.5	42	60.5	06	63.2	38	65.	59	69.3	49
20	283.3	15.4	15.7	16.1	51.6	25	52.3	33	53.2	44	55.8	15	58.3	45	60.2	09	63.	41	64.8	13 02	69.	53
30	281.	15.3	15.7	16.	51.4	27	52.1	36	53.	47	55.6	18	58.1	48	60.	12	62.7	44	64.5	06	68.7	56
40	278.7	15.2	15.6	16.	51.2	29	51.9	38	52.8	49	55.4	21	57.8	51	59.7	15	62.4	47	64.2	09	68.4	14
50	276.5	15.2	15.5	15.7	51.	32	51.7	41	52.6	52	55.2	23	57.6	53	59.5	18	62.2	50	64.	12	68.2	03

TABLE I.—*Continued.*

Degree of Curvature of Divergence.	Corresponding Radius.	Distance from Origin to Head-Block.			Distances of Frog from Origin, and Frog-Angles. When a or $h-a=$																	
					4.7		4.83		5.0		5.5		6.0		6.4		7.0		7.4		8.4	
		$t=$ 0.42	$t=$ 0.44	$t=$ 0.46	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.
21° ′	274.4	15.1	15.5	15.8	50.8	10°35′	51.5	10°43′	52.4	10°54′	54.9	11°26′	57.4	11°56′	59.3	12°21′	62.	12°53′	63.7	13°15′	67.9	14°06′
10	272.3	15.1	15.4	15.8	50.6	37	51.3	46	52.2	57	54.7	29	57.2	59	59.	23	61.7	56	63.4	18	67.6	10
20	270.2	15.	15.4	15.7	50.4	39	51.1	48	52.	59	54.5	31	56.9	12 02	58.8	26	61.5	59	63.2	21	67.3	13
30	263.1	14.9	15.3	15.6	50.2	42	50.9	51	51.8	11 02	54.3	34	56.7	05	58.6	29	61.3	13 02	62.9	24	67.1	16
40	266.	14.9	15.2	15.6	50.	44	50.7	53	51.6	04	54.1	37	56.5	08	58.3	32	61.	05	62.7	27	66.8	19
50	264.	14.8	15.2	15.5	49.8	47	50.5	56	51.4	07	53.9	39	56.3	11	58.1	35	60.8	08	62.4	30	66.6	22
22	262.	14.8	15.1	15.5	49.6	49	50.3	58	51.2	09	53.7	42	56.1	14	57.9	38	60.6	11	62.2	33	66.3	26
10	260.1	14.7	15.1	15.4	49.4	52	50.1	11 01	51.	12	53.5	45	55.9	16	57.7	41	60.3	14	62.	36	66.1	29
20	258.2	14.7	15.	15.3	49.3	54	49.9	03	50.8	14	53.3	47	55.7	19	57.5	44	60.1	17	61.7	39	65.8	32
30	256.3	14.6	14.9	15.3	49.1	56	49.7	05	50.6	17	53.1	50	55.4	21	57.3	46	59.9	20	61.5	42	65.6	36
40	254.4	14.6	14.9	15.2	48.9	59	49.6	08	50.4	19	52.9	52	55.2	24	57.	49	59.7	23	61.3	45	65.3	39
50	252.6	14.5	14.8	15.2	48.7	11 01	49.4	10	50.2	22	52.7	53	55.	27	56.8	52	59.5	26	61.1	48	65.1	42
23	250.8	14.5	14.8	15.1	48.6	03	49.2	13	50.1	24	52.5	57	54.9	29	56.7	54	59.3	28	60.9	51	64.9	45
10	249.	14.4	14.7	15.1	48.4	06	49.	15	49.9	27	52.3	12	54.7	32	56.4	57	59.	31	60.7	54	64.7	48
20	247.3	14.4	14.7	15.	48.2	08	48.9	17	49.7	29	52.2	03	54.5	34	56.2	13	58.8	34	60.5	57	64.5	51
30	245.5	14.3	14.6	15.	48.	11	48.7	20	49.6	31	52.	05	54.3	37	56.	02	58.6	37	60.3	14	64.3	54
40	243.8	14.2	14.6	14.9	47.9	13	48.5	22	49.4	34	51.8	08	54.1	40	55.8	05	58.4	40	60.1	03	64.	57
50	242.1	14.2	14.5	14.9	47.7	15	48.4	25	49.2	36	51.6	10	53.9	42	55.6	08	58.2	43	59.9	05	63.8	15
24	240.5	14.1	14.5	14.8	47.6	18	48.2	27	49.	39	51.4	13	53.7	45	55.5	11	58.	45	59.6	08	63.6	03
10	238.9	14.1	14.4	14.8	47.4	20	48.	29	48.9	41	51.3	15	53.5	47	55.3	14	57.8	48	59.4	11	63.4	06
20	237.3	14.	14.4	14.7	47.2	22	47.9	31	48.7	43	51.1	18	53.4	50	55.2	16	57.6	51	59.3	14	63.2	09
30	235.7	14.	14.3	14.7	47.1	24	47.7	34	48.5	46	50.9	20	53.2	53	55.	18	57.4	54	59.1	17	62.9	12
40	234.1	14.	14.3	14.6	46.9	27	47.6	36	48.4	48	50.8	22	53.	55	54.8	21	57.2	57	58.9	20	62.7	15
50	232.5	13.9	14.2	14.6	46.7	29	47.4	38	48.2	50	50.6	25	52.8	58	54.6	24	57.1	59	58.7	23	62.5	18

TABLE I.—*Continued.*

Degree of Curvature or Divergence.	Corresponding Radius.	Distance from Origin to Head-Block.			Distances of Frog from Origin, and Frog-Angles. When a or $h-a$ =																	
					4.7		4.83		5.0		5.5		6.0		6.4		7.0		7.4		8.4	
		$t=$ 0.42	$t=$ 0.44	$t=$ 0.46	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.
25° ′	231.	13.9	14.2	14.5	46.6	11° 31′	47.2	11° 41′	48.1	11° 53′	50.4	12° 27′	52.6	13° ′	54.4	13° 26′	56.9	14° 02′	58.5	14° 26′	62.3	15° 21′
10	229.5	13.8	14.1	14.5	46.5	33	47.1	43	47.9	55	49.3	30	52.5	03	54.2	29	56.7	05	58.3	29	62.1	24
20	228.	13.8	14.1	14.4	46.3	36	46.9	45	47.8	57	49.1	32	52.3	05	54.1	32	56.5	07	58.1	31	61.9	27
30	226.6	13.7	14.	14.4	46.2	38	46.8	47	47.6	12	48.9	35	52.1	08	53.9	35	56.3	10	57.9	34	61.7	30
40	225.1	13.7	14.	14.3	46.	40	46.6	50	47.5	02	48.8	37	52.	10	53.7	37	56.2	13	57.7	37	61.5	33
50	223.7	13.6	14.	14.3	45.9	42	46.5	52	47.3	04	48.6	39	51.8	13	53.5	39	56.	15	57.5	40	61.3	36
26	222.3	13.6	13.9	14.2	45.7	44	46.3	54	47.2	06	49.5	42	51.6	15	53.3	42	55.8	18	57.3	43	61.1	39
10	220.9	13.5	13.9	14.2	45.6	47	46.2	56	47.	09	49.3	44	51.5	18	53.2	44	55.6	21	57.1	45	60.9	42
20	219.5	13.5	13.8	14.1	45.4	49	46.	59	46.8	11	49.1	47	51.3	20	53.	47	55.4	23	57.	48	60.7	45
30	218.1	13.5	13.8	14.1	45.3	51	45.9	12 01	46.7	13	49.	49	51.2	22	52.9	49	55.3	26	56.8	50	60.5	48
40	216.8	13.4	13.7	14.	45.1	53	45.8	03	46.6	15	48.8	51	51.	25	52.7	52	55.1	29	56.6	53	60.3	51
50	215.5	13.4	13.7	14.	45.	55	45.6	05	46.4	18	48.7	53	50.8	27	52.5	54	54.9	31	56.5	56	60.2	54
27	214.2	13.3	13.6	14.	44.9	58	45.5	07	46.3	20	48.5	56	50.7	30	52.4	57	54.8	34	56.3	58	60.	56
10	212.9	13.3	13.6	13.9	44.7	12	45.3	10	46.1	22	48.4	58	50.5	32	52.2	59	54.6	37	56.1	15	59.8	59
20	211.6	13.3	13.6	13.9	44.6	02	45.2	12	46.	24	48.2	13	50.4	35	52.1	14 01	54.4	39	56.	03	59.6	16 02
30	210.3	13.2	13.5	13.8	44.5	04	45.1	14	45.9	27	48.1	03	50.2	37	51.9	04	54.2	42	55.8	06	59.4	05
40	209.1	13.2	13.5	13.8	44.3	06	45.	16	45.7	29	48.	05	50.1	40	51.8	07	54.1	45	55.6	09	59.2	08
50	207.9	13.1	13.4	13.7	44.2	08	44.8	18	45.6	31	47.8	07	50.	42	51.6	09	53.9	47	55.5	11	59.1	11
23	206.7	13.1	13.4	13.7	44.1	10	44.7	20	45.5	33	47.7	10	49.8	44	51.5	12	53.8	50	55.3	14	58.9	13
10	205.5	13.1	13.4	13.7	44.	13	44.6	23	45.4	35	47.5	12	49.7	47	51.3	14	53.6	52	55.2	16	58.8	16
20	204.3	13.	13.3	13.6	43.8	15	44.4	25	45.2	37	47.4	14	49.5	49	51.2	17	53.5	55	55.	19	58.6	19
30	203.1	13.	13.3	13.6	43.7	17	44.3	27	45.1	40	47.3	17	49.4	51	51.	19	53.3	57	54.8	22	58.5	22
40	202.	12.9	13.2	13.5	43.6	19	44.2	29	45.	42	47.1	19	49.2	54	50.9	21	53.2	15	54.7	24	58.3	25
50	200.8	12.9	13.2	13.5	43.5	21	44.1	31	44.8	44	47.	21	49.1	56	50.7	23	53.	02	54.5	27	58.1	28

TABLE I.—*Continued.*

Degree of Curvature or Divergence.		Corresponding Radius.	Distance from Origin to Head-Block.			Distances of Frog from Origin, and Frog-Angles. When a or $h-a=$																	
						4.7		4.83		5.0		5.5		6.0		6.4		7.0		7.4		8.4	
			$t-0.42$	$t-0.44$	$t-0.46$	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.	D.	F.
29°	′	199.7	12.9	13.2	13.5	43.3	12° 23′	43.9	12° 33′	44.7	12° 46′	46.9	13° 23′	49.	13° 59′	50.6	14° 26′	52.9	15° 05′	54.4	15° 30′	57.9	16° 30′
	10	198.6	12.8	13.1	13.4	43.2	25	43.8	35	44.6	48	46.7	26	48.8	14 01	50.4	28	52.7	07	54.2	32	57.8	33
	20	197.5	12.8	13.1	13.4	43.1	27	43.7	37	44.4	50	46.6	28	48.7	03	50.3	31	52.6	10	54.1	35	57.6	36
	30	196.4	12.8	13.1	13.4	43.	29	43.6	39	44.3	52	46.5	30	48.6	06	50.2	33	52.4	12	53.9	38	57.4	39
	40	195.3	12.7	13.	13.3	42.8	31	43.4	41	44.2	55	46.4	32	48.4	08	50.	35	52.3	15	53.8	41	57.3	41
	50	194.2	12.7	13.	13.3	42.7	33	43.3	44	44.1	57	46.3	34	48.3	10	49.9	37	52.1	17	53.6	44	57.1	44
30		193.2	12.7	13.	13.2	42.6	35	43.2	46	44.	59	46.1	36	48.2	12	49.7	40	52.	20	53.5	46	57.	47
	10	192.2	12.6	12.9	13.2	42.5	37	43.1	48	43.8	13 01	46.	39	48.	15	49.6	42	51.8	22	53.3	49	56.8	49
	20	191.1	12.6	12.9	13.2	42.3	39	43.	50	43.7	03	45.9	41	47.9	17	49.4	44	51.7	25	53.2	51	56.7	52
	30	190.1	12.5	12.8	13.1	42.2	41	42.9	52	43.6	05	45.8	43	47.8	19	49.3	47	51.6	27	53.	53	56.5	54
	40	189.1	12.5	12.8	13.1	42.1	43	42.8	54	43.5	07	45.6	45	47.6	22	49.2	49	51.5	30	52.9	56	56.4	57
	50	188.1	12.4	12.7	13.	42.	45	42.7	56	43.4	09	45.5	47	47.5	24	49.	51	51.3	32	52.7	58	56.2	17
31		187.1	12.4	12.7	13.	41.9	47	42.6	58	43.3	11	45.4	49	47.4	26	48.9	54	51.2	35	52.6	16 01	56.1	03
	30	184.3	12.3	12.6	12.9	41.6	53	42.2	13 04	43.	17	45.	56	47.	32	48.5	15 01	50.8	42	52.2	09	55.6	11
32		181.4	12.2	12.5	12.8	41.3	59	41.9	10	42.6	23	44.7	14 02	46.7	39	48.2	08	50.4	49	51.8	16	55.2	18
	30	178.7	12.2	12.4	12.7	41.	13 05	41.6	16	42.3	29	44.3	09	46.3	45	47.9	15	50.	56	51.4	23	54.8	26
33		176.1	12.1	12.3	12.6	40.7	11	41.3	22	42.	35	44.	15	46.	52	47.5	21	49.7	16 03	51.	30	54.4	34
	30	173.5	12.	12.2	12.5	40.4	17	41.	27	41.7	46	43.7	21	45.6	59	47.2	28	49.3	10	50.6	37	54.	42
34		171.	11.9	12.2	12.4	40.1	22	40.7	33	41.4	47	43.4	27	45.3	15 05	46.8	35	49.	17	50.3	44	53.6	49
	30	168.6	11.8	12.1	12.3	39.8	28	40.5	39	41.1	53	43.1	33	45.	12	46.5	42	48.7	24	50.	51	53.2	57
35		166.3	11.7	12.	12.3	39.5	34	40.2	45	40.8	59	42.8	39	44.7	18	46.2	48	48.3	31	49.6	58	52.9	18 04
	30	164.	11.6	11.9	12.2	39.3	40	39.9	51	40.5	14 05	42.5	45	44.4	24	45.8	55	48.	38	48.2	17 05	52.5	12
36		161.8	11.6	11.8	12.1	39.	45	39.6	57	40.2	10	42.2	51	44.1	30	45.5	16 01	47.6	44	48.9	12	52.2	20
	30	159.6	11.5	11.7	12.	38.7	51	39.3	14 03	40.	15	42.	57	43.8	36	45.2	07	47.3	51	48.6	10	51.7	27